W26. If a, b, c are strictly positive real numbers, then:

$$(a^a b^b c^c)^{\frac{1}{a+b+c}} + (a^b b^c c^a)^{\frac{1}{a+b+c}} + (a^c b^a c^b)^{\frac{1}{a+b+c}} \le a+b+c$$

Dorin Mărghidanu.

Solution by Arkady Alt, **San Jose**, **California**, **USA**.

Applying consecutively weighted AM-GM Inequality $a^p b^q c^r \le pa + qb + rc$, where $a, b, c, p, q, r > 0$ and $p + q + r = 1$ we obtain

$(a^a b^b c^c)^{\frac{1}{a+b+c}} \le \dfrac{a^2+b^2+c^2}{a+b+c}, (a^b b^c c^a)^{\frac{1}{a+b+c}} \le \dfrac{ba+cb+ac}{a+b+c}$,

$(a^c b^a c^b)^{\frac{1}{a+b+c}} \le \dfrac{ca+ab+bc}{a+b+c}$ and, therefore,

$$(a^a b^b c^c)^{\frac{1}{a+b+c}} + (a^b b^c c^a)^{\frac{1}{a+b+c}} + (a^c b^a c^b)^{\frac{1}{a+b+c}} \le \frac{(a+b+c)^2}{a+b+c} = a+b+c.$$